

# Power Flow Tracing

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Analyzing the Embedding of Eleks Dakar  
in Research and Practice

FRAUNHOFER-INSTITUTE FOR APPLIED INFORMATION TECHNOLOGY FIT  
BRANCH BUSINESS & INFORMATION SYSTEMS ENGINEERING

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in Research and Practice

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# 1

## Motivation

# Motivation

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/ The energy market is undergoing a profound transformation, among other things, driven by deregulation and liberalization (e.g., in the EU since 1996 with the first European Directive 96/92/EC) as well as an increasing emphasis on the integration of renewable energies.

/ By enabling consumers to choose their suppliers (Directive 2003/54/EC and Directive 2003/55/EC), the **deregulation and liberalization of the European energy market** have created a competitive environment. This results in the need to develop reliable methods for enabling a fair cost allocation among market participants (e.g., related to grid usage and losses) and to provide transparency about energy flows to stakeholders (e.g., for more informed decision-making). Regarding electricity, for example, tracing the exact path that physical current will take from producer to consumer to allocate costs accordingly is inherently complex due to the physical characteristics of the grid and the nature of electricity flows: unlike other commodities, electricity cannot be dyed or tagged to visually trace its path through the grid. This characteristic presents unique challenges and necessitates the use of sophisticated methods to approximate flow paths.

/ The shift toward increased **integration of renewable energy sources**, characterized by a large number of decentralized small-scale plants of producers and prosumers (i.e., consumers that can also act as producers), leads to a complex grid with more active participants. This further complicates an accurate and transparent allocation of costs. It also makes the distinction between electricity from renewable and non-renewable energy sources (e.g., for regulatory compliance, competitive advantages, or environmental reasons) equally complex.

/ To date, many methods for allocating costs (e.g., the postage stamp method) or renewable origin (e.g., Guarantees of Origin) of electricity have disregarded the physical grid restrictions and trace electricity as a commodity “on the balance sheet”, which can result in issues relating to fairness and credibility.

/ In this light, several physical tracing approaches have emerged under the collective term **Power Flow Tracing (PFT)**. PFT approaches aim to map the paths that electricity will take from generators to loads using algorithms, often applying basic physical laws as well as assumptions. By doing so, PFT approaches promise to enable an accurate allocation of the share that individual generators or loads have on electricity production, consumption, and line losses. Such an accurate allocation can be crucial for a variety of stakeholders in a plethora of application areas. Utilities, regulators, and energy traders, for instance, rely on detailed flow information to optimize network management, inform policy development, and guide market operations. Consumers and producers, on the other hand, can benefit from the transparency and fairness in electricity distribution that PFT may facilitate, thereby possibly promoting trust and enhancing market efficiency.

/ Our comparative analysis aims to provide a general understanding of the strengths and limitations of PFT in general and to give an overview of individual methods, with a particular focus on Eleks Dakar PFT. By highlighting the theoretical foundations and practical application areas of these PFT methods, we provide insights into their suitability in different contexts. By analyzing the scientific embedding of Eleks Dakar PFT and comparing this approach with other methods currently in practice, we aim to assess its effectiveness and explore its potential for wider adoption in the energy sector. This analysis not only contributes to academic knowledge, but also provides practical insights for industry stakeholders, such as those seeking to improve grid management and transparency.

## Power Flow Tracing

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Power Flow Tracing (PFT) is a collective term for a set of methods that allow the calculation of the power transfers from individual generators to individual loads and branches.

Originally established to allocate costs for transmission losses, PFT recently gained a lot of attraction from research and practice for further use cases, such as emissions allocation.



# 2

## Foundations

# Foundations



/ In the traditional power supply chain, power flows unidirectionally from generation to consumption, with centralized systems for generation, transmission, and distribution. However, the increasing share of renewables in the electricity mix is driving a shift toward a more decentralized system. This decentralization is characterized by an increase in distributed generation, which is predominantly renewable, small-scale, and located close to the point of consumption. As a result, tracing the origin and path of electricity is becoming increasingly complex. In terms of this tracing, it is important to distinguish between the physical path of electricity and its balance sheet representation, as illustrated in a simplified way in Figure 1.

## 2.1 Tracing of Electricity on the Balance Sheet and Physically

/ The **balance sheet path** treats electricity as a tradable commodity. Typically, a supplier enters contracts with producers to secure capacity and then purchases the electricity. Any discrepancies in capacity are balanced out on the energy market (e.g., power exchange). The electricity is then sold to customers. By the end of a billing period, it is possible to determine which producers delivered how much electricity from which sources to the suppliers, and how the suppliers distributed this electricity to their customers (Körner et al. 2024).

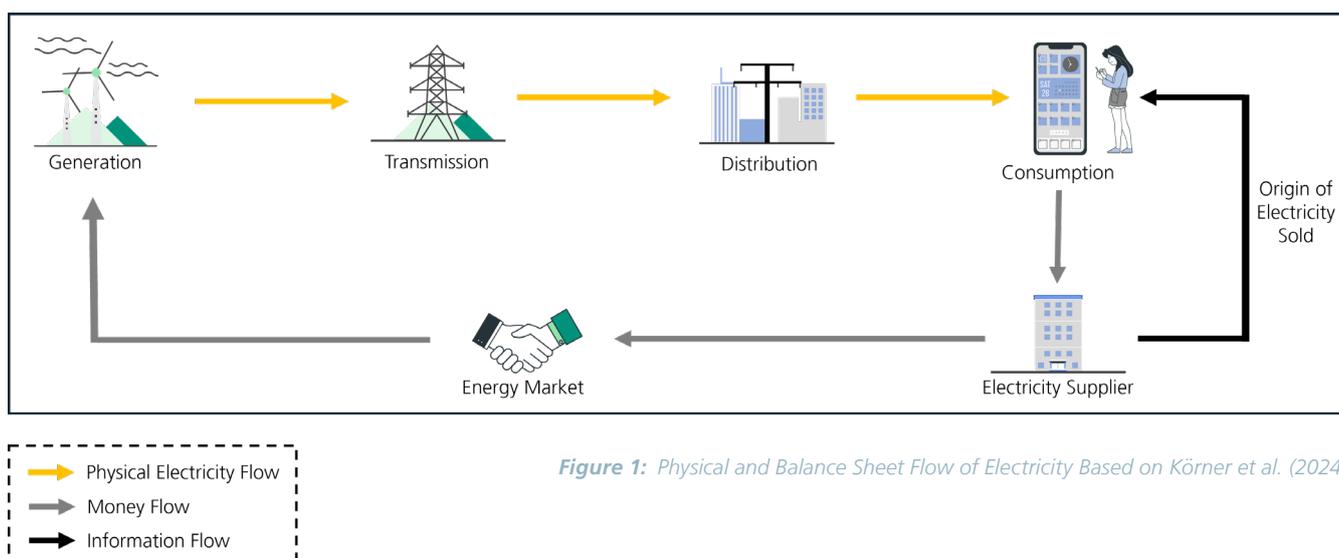


Figure 1: Physical and Balance Sheet Flow of Electricity Based on Körner et al. (2024)

/ Balance sheet accounting allows for the certification of electricity independently of its physical flow, among others, enabling it to be counted as renewable energy and contributing to decarbonization goals. The certification frameworks in place in Europe, such as Guarantees of Origin and Renewable Energy Certificates, provide mechanisms for tracing and certifying electricity and its characteristics. Current certification schemes are, however, completely decoupled from the physical power flow and lack temporal and local granularity (Babel et al. 2024).

/ The **physical path** refers to the actual flow of electricity through the grid. In this process, electricity generated by plants is fed from the transmission grid into the local distribution grid, from which consumers receive it. Managing the balance of supply and demand within this network is crucial, as Kirchhoff's laws for electrical circuits (cf. Chapter 2.2) hold. Local balancing is typically handled by balancing group managers, such as municipal utilities, while transmission system operators manage it at a supra-regional level. A key characteristic of the physical path is the indistinguishability of electricity once it enters the grid due the inability to "dye" power flows. PFT approaches aim to trace power despite this circumstance, often based on assumptions such as proportional sharing.

## 2.2 Kirchhoff's Laws

/ In order to describe the physical path as accurate as possible, PFT approaches typically respect the fulfillment of the laws provided by Kirchhoff (1845). These laws are fundamental to the description of the behavior of voltage and current in electric circuits and include the following (cf. Figure 2):

1. Junction Rule: The current flowing into a node (junction) equals the current flowing out of it
2. Loop Rule: In a complete loop, the sum of all voltages around this loop equals zero

/ In the subsequent chapters, we will explore tracing of the physical path in more detail, thereby describing different PFT methods, their underlying assumptions, and their applications. The balance sheet approaches and a concept that aims at addressing their current shortcomings related to emissions in the electricity sector are described in our white paper [Digital Proofs of Origin for Sustainability - Assessing a Digital Identity-Based Approach in the Energy Sector](#).

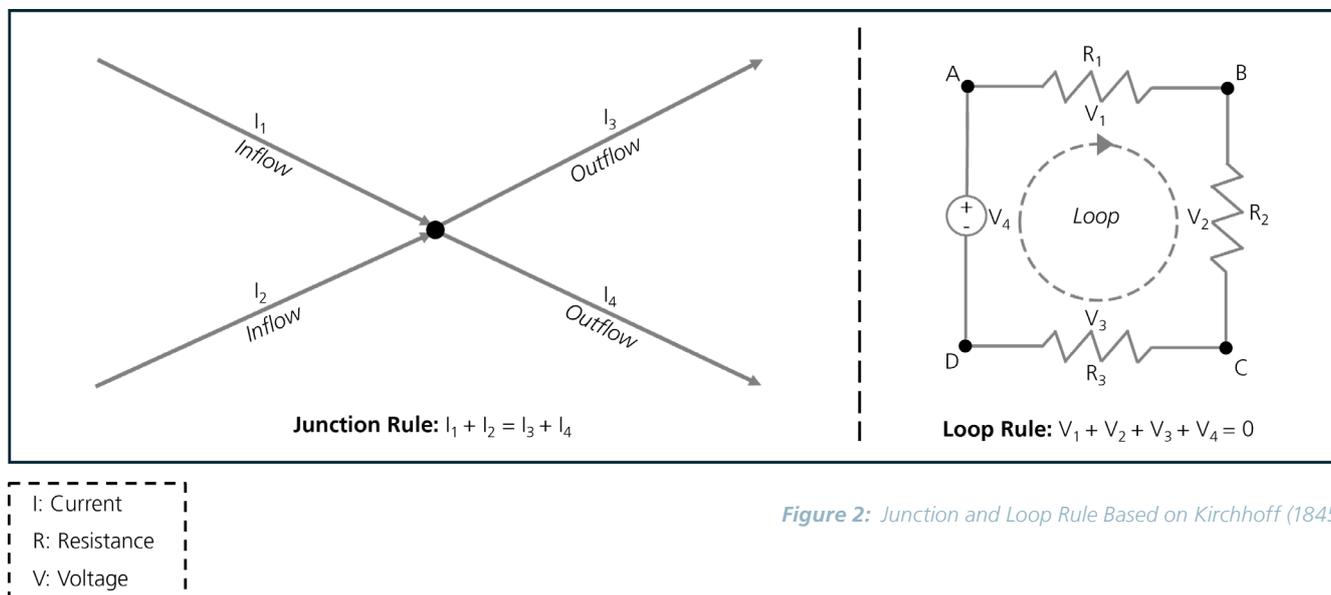
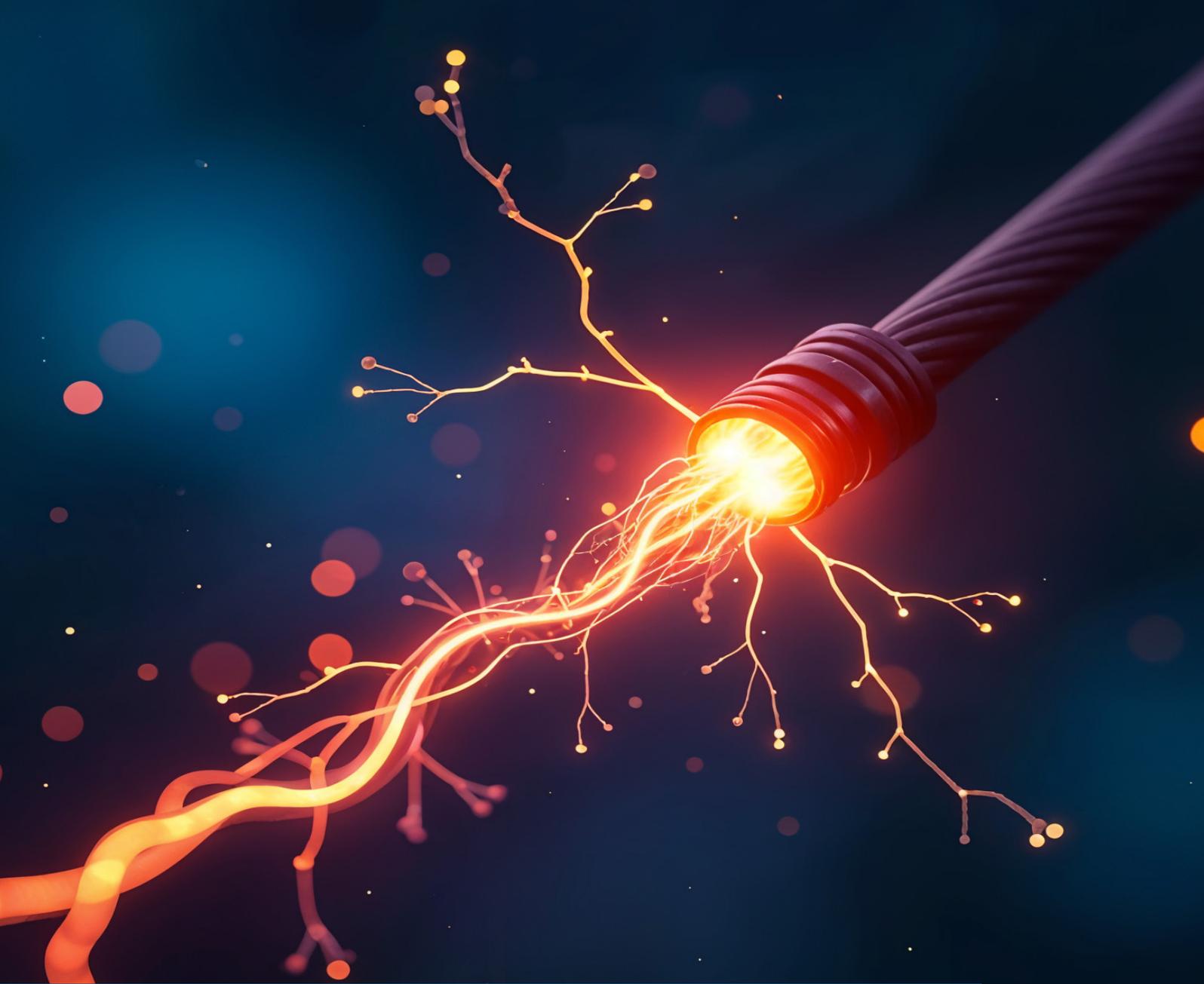


Figure 2: Junction and Loop Rule Based on Kirchhoff (1845)



# 3

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## Eleks Dakar Power Flow Tracing

# Eleks Dakar Power Flow Tracing



/ Eleks GmbH is an international software company that provides software development, product design, quality assurance, and consultancy services. Among other things, they developed a tool for modeling, analysis, planning, and optimization of electrical networks called Eleks Dakar. In this section, we formally describe the PFT method of Eleks Dakar in order to provide a general understanding and a basis for analyzing its embedding in research and practice. We derived this description from public information as well as internal documentation provided by Eleks GmbH, including a white paper and source code. The Eleks Dakar team was consulted to clarify specific details. To protect intellectual property, the descriptions here are generalized and have been approved by Eleks GmbH for public disclosure.

## 3.1 Method Based on Proportional Sharing

/ The main purpose of Eleks Dakar PFT is to accurately allocate the contributions of generators to the line flows and loads within a network. The method revolves around the proportional sharing principle, as outlined by Bialek (1996). This principle states that power flows converging at a node (junction) are proportionally divided among the outgoing branches based on their respective contributions (cf. Figure 3).

/ This assumption makes it possible to calculate how much power each generator contributes to different loads. In the example illustrated in Figure 3, node n acts as a “perfect mixer” of the power coming from generators j and k. Loads m and l each receive  $\frac{30MW}{30MW+70MW} = 30\%$  of their power from generator j (and, respectively, 70% from generator k). In other words, load m obtains 18 MW from generator j and 42 MW from generator k according to the proportional sharing principle (load l 12 MW from j and 28 MW from k, respectively).

/ According to Eleks GmbH, their solution of the power allocation problem based on the proportional sharing does not depend on the state of the network and is widely used to allocate the costs of electric energy transmission. Further, they state it enables determining the following:



- A participation share of every power station in load supply
- Power flows that run from every generator in the branches of an equivalent circuit of an electrical network
- Power losses occurring while transmitting load from generation to every load «

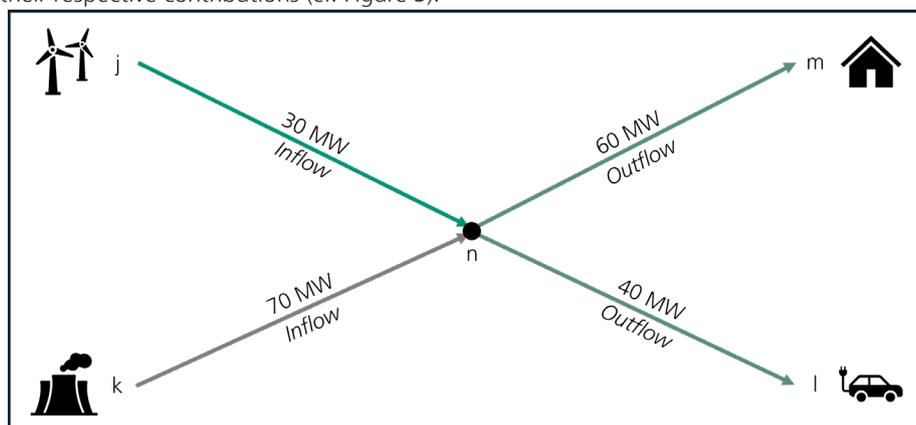


Figure 3: Proportional Sharing Principle Based on Bialek (1996)

### 3.2 Tracing Based on the Result of the Calculation of the Power Flow

/ In order to perform PFT based on the proportional sharing principle, Eleks GmbH considers the results of balanced power flow calculations (i.e., calculations for determining characteristics of a given power system in its steady state such as generation and load at buses) as essential. This requires comprehensive information about the network (i.e., about generation and load requirements, transformer and line parameters, and voltage). Based on this information, a power flow result can be obtained through solving a set of equations. Eleks Dakar PFT derives the power flow using a modified node-by-node Newton’s method, thereby referring to Skrypnik and Konoval (2011). According to Eleks GmbH, the approach follows Kirchhoff’s circuit laws (cf. Chapter 2.2) and is particularly effective for analyzing marginal or weighted states in nonlinear systems. The result of the power flow calculations is used as input for a topological PFT approach. Instead of relying on the linear equation-based method of Bialek (1996), which requires the creation and inversion of matrices, Eleks Dakar PFT relies on the graph-based method outlined by Kirschen et al. (1997). Eleks GmbH argues with a reduced computational intensity as well as the convenience of the representation of the power flows. They slightly adapt the original approach to effectively handle empty nodes (i.e., nodes with no load or generation).

### 3.3 Implementation

/ Eleks Dakar aims to provide a numerical result for PFT as well as a corresponding visualization. To do so, the method iteratively computes the individual nodes and branches according to the approach described above and returns the tracing results after all nodes have been analyzed. While the exact implementation is the intellectual property of ELEKS GmbH and is therefore not disclosed here, the general procedure is similar to that formulated in the foundational paper by Kirschen et al. (1997); there are two algorithms, one for tracing from a power source to a load (i.e., downstream) and one for tracing from a load to a power source (i.e., upstream). Since the two algorithms work similarly, we briefly describe the former as an example below.

/ After receiving the results of the power flow calculations, one node at a time is selected, and for that node, the branches with power flowing from it are analyzed one by one. The results of the power flow calculations make it possible to define allocation ratios for each branch and, ultimately, for the node. After all branches of a node have been analyzed, the next node is selected, and the process is repeated. The high-level process for downstream tracing from source to load is illustrated in Figure 4.

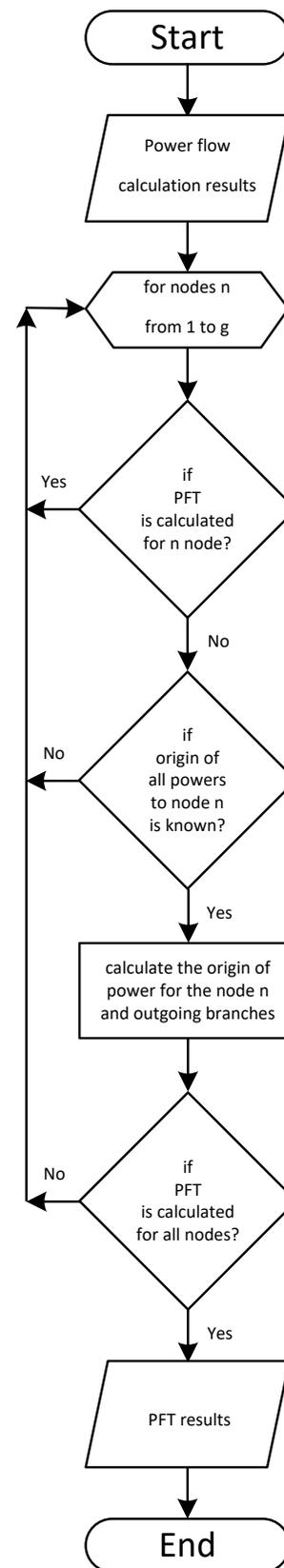


Figure 4: Simplified Illustration of the Eleks Dakar PFT process



# 4

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## Scientific Embedding of Eleks Dakar

# Scientific Embedding of Eleks Dakar

/ In this chapter, we analyze the scientific embedding of Eleks Dakar PFT by analyzing the foundational literature and claims made by Eleks GmbH (cf. Chapter 3). We also contextualize their method with others in peer-reviewed, academic PFT literature. To do so, we conduct a literature review, based on which we map and compare the different PFT methods that we find. Appendix A illustrates the final set of publications that we include for this comparison. This set corresponds to our results after applying objective criteria (recency, language, review-process) and subjective criteria (quality, relevance) to our initial set of around 1 000 publications which we analyzed iteratively based on title, abstract, and full text. In addition to

this set, we include foundational papers that are mentioned in relevant works and by Eleks GmbH. PFT methods can be broadly categorized into seven approaches: linear equations, graph theory, game theory, optimization, circuit theory, relative electrical distance, and equilateral bilateral exchange. They vary mainly in the arithmetic and how they treat loop flows, losses, and reactive power (Tijani et al. 2019). We particularly shed light on methods that follow the proportional sharing principle (i.e., linear equation-based and graph theory-based methods) as they are the most widely used (cf. Appendix A) and Eleks Dakar employs such a method. We summarize key differences between the different PFT approaches in Table 1 and highlight

#	Method	Basic Tracing Approach	Incorporation of Fairness Conditions	Computation Time*	Exemplary Sources
1	Linear Equations	Proportional Sharing	-	2	Ma et al. (2023), Schäfer et al. (2019)
2	Graph Theory	Proportional Sharing	-	2	Lawal et al. (2019), Yu (2022)
3	Game Theory	Economic Principles	Depending on Objective Function	4	Rao et al. (2010), Zuo et al. (2024)
4	Optimization	Optimization Approach	Depending on Objective Function	4	Abhyankar et al. (2006), Budi et al. (2020)
5	Circuit Theory	Network Matrices	-	3	Chen and Dhople (2020), Lu and Zou (2021)
6	Relative Electrical Distance	Network Matrices	-	3	Visakha et al. (2004), Vlaisavljevic et al. (2019)
7	Equivalent Bilateral Exchange	Proportional Supply of Every Load by Every Generator	-	3	Galiana et al. (2003)

\*minimum: 1, maximum: 5; may deviate depending on the exact approach

Table 1: Comparison of Power Flow Tracing Methods based on Khan and Agnihotri (2013)

## 4.1 Proportional Sharing Based Methods

/ The Eleks Dakar PFT method is grounded in the **proportional sharing principle** based on Bialek (1996) (cf. Chapter 3.1). Most of the recent PFT literature we find is based on this principle and recognizes Bialek's (1996) **linear equation-based** approach as the foundation of PFT (e.g., Bai and Crisostomi (2020), Ma et al. (2022), or Ren et al. (2023)). Based on the proportional sharing principle and the requirement that Kirchhoff's current law (cf. Figure 2) is always satisfied, Bialek (1996) derives two algorithms – one upstream-looking algorithm for the power inflows (i.e., to determine the distribution of power from generation to loads) and one downstream-looking algorithm for the power outflows of a node (i.e., to determine how the demand of the load is satisfied by generators) (De and Goswami 2010). The former not only allows for the identification of the contribution of each generator to meeting specific load demands, but also facilitates the allocation of total transmission losses to individual loads within the network. This allows loads to be charged individually based on the actual amount of power lost. Accordingly, the latter facilitates not only the determination of how the output of a specific generator is distributed among all loads, but also the allocation of the total transmission loss to each generator in the network (Bialek 1996). In accordance with the claims made by Eleks Dakar, this allows to derive an individual participation share and the charging of every generator and load for their transmission losses. For the concrete mathematical operations we refer to the original work of Bialek (1996), as they exceed the scope of this study.

/ As stated by Eleks Dakar, Bialek (1996) works on the results of a power flow calculation (or a state estimation) to determine the characteristics of the power system. While the scope of this study does not allow for a detailed analysis of power flow calculation methods, we note that many recent publications, similarly to Eleks Dakar, rely on a Newton's approach (e.g., Lawal et al. (2019), Li et al. (2023), or Zhang et al. (2023a)). Further, the approach of Bialek (1996) is topological (i.e., it addresses a general transportation problem of how flows are distributed). As such an approach does not inherently consider transmission losses and can, hence, not account for reactive power flows, Bialek (1996) first derives lossless flows. As a simple way to do so, he suggests deriving an average line flow by adding half the line loss to the inflow at the terminal node of that line. Fictitious nodes can also be added as additional sources or sinks to represent losses. However, this increases the computational complexity, which Pantos et al. (2005) have addressed by using matrix decoupling.

/ Another approach to consider losses and reactive flows is relying on nodal distribution factors instead of topological ones, as introduced by Grgic and Gubina (2000). A further variation of the linear equation-based method includes Abdelkader (2007), who considers active and reactive power flows as well as complex losses simultaneously, does not need determination of the feed paths between generation and loads, and neither requires matrix inversion nor additional nodes for representing losses.

/ Based on the proportional sharing principle, approaches have been developed that are not based on linear equations like Bialek (1996) but instead rely on **graph theory**. Kirschen et al. (1997) provide the first scientific publication on a graph-based PFT method. To do so, they divide several buses into sets of contiguous buses that are each fed from the same source ("commons") that are connected by branches ("links") and can be represented as directed graphs. The method is also applicable to active and reactive power flows and was initially suggested for geographically differentiated spot pricing, pricing of transmission services, loss allocation, and visualization for operators to get a better understanding of the state of the power system (Kirschen et al. 1997). Acha (2007) provides a slight variation of this approach, in which source dominions (i.e., directed graphs consisting of one source and one or multiple sinks) and common branches (i.e., branches that belong to the same dominion) are used instead of commons and links. Other approaches include De and Goswami (2010), who provide a PFT based loss allocation method that does, in contrast to other graph theoretic approaches, not require sub-grouping of generator or load buses, making it very simple.

### Proportional Sharing Assumption

While researchers have made arguments to justify the proportional sharing principle (e.g., based on game theory, information theory, or the maximum entropy principle (Bialek and Kattuman 2004)) and it is generally seen as intuitive, it remains an assumption that can neither be proven nor disproven (Vega-Fuentes et al. 2022; Wang et al. 2022; Wu et al. 2019).

/ Research in power engineering suggests that linear equation- and graph-based methods based on the proportional sharing principle are the same at their core and deliver the same results. Ansyari et al. (2007), for example, show this by comparing the approaches of Acha (2007), Bialek (1996), and Kirschen et al. (1997). Achayuthakan et al. (2010) further provide a mathematical representation that proves the link between the linear equation- and graph-based methods. While all methods technically require matrix inversion, the matrix does not necessarily need to be explicitly formed in the graph-based methods. As stated above, this is also the case for the Abdelkader (2007) method, suggesting that the choice between these methods can be made according to personal preference. Thus, we consider the graph theory approach of Eleks GmbH based on Kirschen et al. (1997) to be reasonable but note that there are indeed also linear equation-based methods that do not require explicit matrix inversion.

## 4.2 Other Methods

/ Consumers do not necessarily have a choice about where their physical electricity comes from, nor do generators have control over where their generated electricity goes (Kirschen et al. 1997). Hence, the proportional sharing principle raises questions related to **fairness**, e.g., regarding price allocation. Against this background, there are methods that apply **game theory** to PFT. Game theory is a mathematical theory that allows the modeling of decision-making situations in which multiple participants are interacting with each other. In particular, cooperative game theory can be used to address the problem of allocating network losses caused by interactions between entities in an electricity market by considering the impact of transactions on network losses (Zuo et al. 2024). For example, Rao et al. (2010) apply cooperative game theory to PFT to achieve a »min-max fair« tracing solution (i.e., any reduction in, say, the unit cost of one entity computed in the tracing framework leads to an increase in the unit cost of another entity that must pay either the same or a higher unit cost).

/ Similarly, there are approaches that see PFT as an **optimization** problem. For example, Abhyankar et al. (2006) do so by focusing on transmission costs. They model the solution space of possible tracing solutions and formulate an optimal tracing problem with linear constraints. In their case, the formulation of the problem aims to derive a PFT-compliant solution that is as close as possible to the "postage stamp" method, a simple and widely used method for allocating transmission losses that does not distinguish between the degree of use of transmission facilities and instead assumes the same network usage per MW for every generator or load (Abhyankar et al. 2006). By doing so, this approach aims to provide a compromise between the proportional sharing principle and the postage

stamp method, and thus to achieve greater fairness.

/ In order to avoid relying on **unverifiable assumptions** like proportional sharing, Chen and Dhople (2020) argue for a PFT method that is based on and consistent with the circuit laws that underlie the steady-state behavior of power systems. **Circuit theory** based approaches mainly originate from the work of Conejo et al. (2001) and rely on the fact that any electric network can be represented as an equivalent circuit (Wang et al. (2022). Chen and Dhople (2020), for example, provide a circuit theory-based method that considers disaggregation for complex power injections in the network: for downstream tracing, the complex power injected by a generator is decomposed into a sum of parts that are attributed to loads and losses in the network. For upstream tracing, the complex power consumed by a load is similarly decomposed, attributed to generators, and allocated to losses. Circuit-theory based methods are often seen as computationally inefficient because they heavily rely on the use of network matrices (Khan and Agnihotri 2013; Bhand and Debbarma 2021). Also, they face challenges in real power systems due to complexities (e.g., the existence of loop flows) and the need for other assumptions or estimations (e.g., related to voltage/current phasors and internal impedance) (Wang et al. 2022).

/ The **relative electrical distance** method, first introduced by Visakha et al. (2004) for transmission cost allocation, was suggested against the background of the **complexity** associated with existing methods. This method is based on a network matrix that provides the relative locations of loads with respect to generators. The authors also suggest charging additional costs for power contracts that deviate from desired load/generation schedules. The relative electrical distance method, however, does not allow for approximating the contribution of individual generators and loads but rather allocates costs based on (the contractual deviation of) a predefined desired schedule, which is why it may not be classified as a PFT method in a narrow sense. This may also limit its applicability in other application areas (cf. Chapter 5).

/ Galiana et al. (2003) first suggested using **equivalent bilateral exchange** as a method to allocate transmission costs, arguing with the lacking **inclusion of counterflows** (i.e., components in the opposite direction of the net flow in a line). In this method, a fraction of each generation is proportionally assigned to each demand and vice versa, in a way that both of Kirchhoff's laws (cf. Figure 2) are fulfilled. Equivalent bilateral exchange, however, describes a theoretical mathematical concept rather than the physical reality of power grids (Khan and Agnihotri 2013). Further, it has not been subject to current research in the area of PFT according to our literature review (cf. Appendix A).



# 5

## Comparison of Eleks Dakar and Other Approaches

# Comparison of Eleks Dakar and Other Approaches

/ In this chapter, we focus on the applications of PFT to demonstrate its suitability compared to other concepts and practices. To do so, we organize our set of relevant literature (cf. Appendix A) according to application areas. We then analyze, by searching for documents from practitioners, whether PFT approaches in these application areas are already being implemented or whether there are application areas for PFT in practice that are not yet covered in our scientific literature.

/ It is worth noting that some organizations, such as Siemens (2024), provide services and functionalities that could include PFT, but do not disclose detailed descriptions or mention PFT specifically. Hence, our overview does not provide a holistic list of all industries and organizations working on PFT but rather a mapping of relevant activities and application areas in research and practice based on publicly available information, as illustrated in Table 2.

Application Area	Suggested PFT Methods (cf. Table 1)	Recent Contributions from Research	Recent Contributions from Practitioners
Cost Allocation and Transparency in Transmission Networks	#1, #2, #3, #4, #5, #6, #7	Enshae and Yousefi (2019), Schäfer et al. (2019), Shuai et al. (2021), Vlaisavljevic et al. (2019)	Electricity Maps (2022), New Zealand Electricity Authority (2015), Tennet TSO B.V (2024)
Cost Allocation and Transparency in Distribution Networks	#1, #2	Budi et al. (2020), Chen and Dhople (2020), Wang et al. (2024), Wanghao and Paul (2019), Yu (2022), Yu et al. (2023), Zhao et al. (2023)	-
Cost Allocation and Transparency in Peer-To-Peer Trading	#1, #2, #5	Bai and Crisostomy (2020), Bhand and Debbarma (2021), Deacon et al. (2021), Lu and Zou (2021)	-
Congestion Management, Curtailment, and Overload Control	#1, #2, #5	Angaphiwatchawal et al. (2024), Jiandong et al. (2019), Jiang and Zhang (2021), Lawal et al. (2019), Wu et al. (2019)	-
Allocation of Renewable Energy and Carbon Emissions	#1, #2, #5	Dudkina et al. (2022; 2024), Li et al. (2023), Liang et al. (2023), Ma et al. (2023), Qing and Xiang (2024), Ren et al. (2023), Wang et al. (2022; 2023), Yan et al. (2021), Yang et al. (2023), Zhang et al. (2023b), Zuo et al. (2024)	50hertz (2023), Electricity Maps (2022), Singularity (2023), University of Freiburg (2024)

Table 2: Overview of Power Flow Tracing Application Areas

## 5.1 Cost Allocation and Transparency in Transmission Networks

/ Cost allocation in transmission networks due to the deregulation and liberalization of the energy market is the core motivation for PFT (cf. Chapter 1). Accordingly, many fundamental papers providing methods for PFT focus on this application area (cf. Chapter 2). Among others, legacy approaches for cost allocation in transmission networks include the postage stamp method outlined above and the contract path method, which bases transmission costs on the most direct physical transmission path (Bai and Crisostomyi 2020). These methods are simple in execution but do not respect the physical paths that electricity may take. PFT approaches can address this issue, as illustrated in Chapter 4. Proportional sharing-based methods are the predominant approach in research and the only approach implemented in practice in this application area.

/ Practitioners that work on PFT in this context include Transmission System Operators, such as the New Zealand Electricity Authority (2015) and Tennet TSO B.V (2024). Further, Electricity Maps (2022) provide a European map that not only visualizes Power Flows, thereby providing more transparency for the public, but also includes an Application Programming Interface (API) for organizations.

## 5.2 Cost Allocation and Transparency in Distribution Networks

/ While initial academic PFT literature focuses on cost allocation in transportation networks, more recent works also consider electricity distribution networks. A major issue that PFT aims to solve in this context is the existence of bidirectional power flows due to the integration of renewable energies and prosumers (cf. Chapter 1) that complicate grid modeling in distribution networks (Wanghao and Paul 2019). As future distribution networks are expected to be increasingly complex and with more dynamic topologies, researchers focus on efficiency (e.g., Zhao et al. (2023) who apply virtual contribution theory to reduce computational complexity), thereby primarily analyzing proportional sharing based methods, as other approaches seem to be less efficient (e.g., circuit theory due to the reliance on network matrices) (Bhand and Debbarma 2021).

/ To the best of our knowledge, there have not yet been any publicly disclosed practical examples of PFT in this application area. This may be due to the large computational complexity associated with PFT in distribution grids. While PFT methods have been implemented in some transmission networks (cf. Chapter 5.1), their feasibility in complex distribution networks

is difficult to validate and may need more testing. Eleks Dakar provided results for a real distribution network, illustrating the potential of their method in this application area.

## 5.3 Cost Allocation and Transparency in Peer-To-Peer Energy Trading

/ Due to the trend of renewable energy integration and the associated shift toward prosumers described above, energy may be directly traded between peers, e.g., to maximize the usage of locally produced electricity. Analogous to Chapter 5.1 and 5.2, losses in such a peer-to-peer energy trading systems can be allocated either by simple approaches that do not consider the physical properties of electricity, such as contract path and postage stamp method, or by PFT approaches to approximate the individual contributions of generators and loads based on electricity flows.

/ Currently implemented approaches only include the former, the reasons for not integrating PFT may include complexity (cf. Chapter 5.2) and the fact that peer-to-peer energy trading systems themselves are not widely applied in energy markets yet.

## 5.4 Congestion Management, Curtailment, and Overload Control

/ Scholars also proposed methods for congestion management, curtailment, and overload control based on PFT. Angaphiwachawal et al. (2024), for example, employ a modified Bialek (1996) method to address voltage impact of local energy markets in distribution grids. Lawal et al. (2019) similarly integrate PFT in congestion management by using the method described by Acha (2010). They leverage PFT to detect the generators contributing to congestion and suggest an output reduction as penalty. Wu et al. (2019) propose a load curtailment method based on an extended incidence matrix. The use of sophisticated PFT methods in this context is, to our knowledge, mainly driven by research and, to our knowledge, not part of implemented practices yet. Existing methods in practice instead rely on estimates or marginal prices, which are both limited in accuracy and calculation time (Angaphiwachawal et al. 2024).

/ PFT has, according to our knowledge, not been applied to congestion management, curtailment, and overload control processes in practice. Reasons may include the need for real-time capabilities and the criticality of these processes.

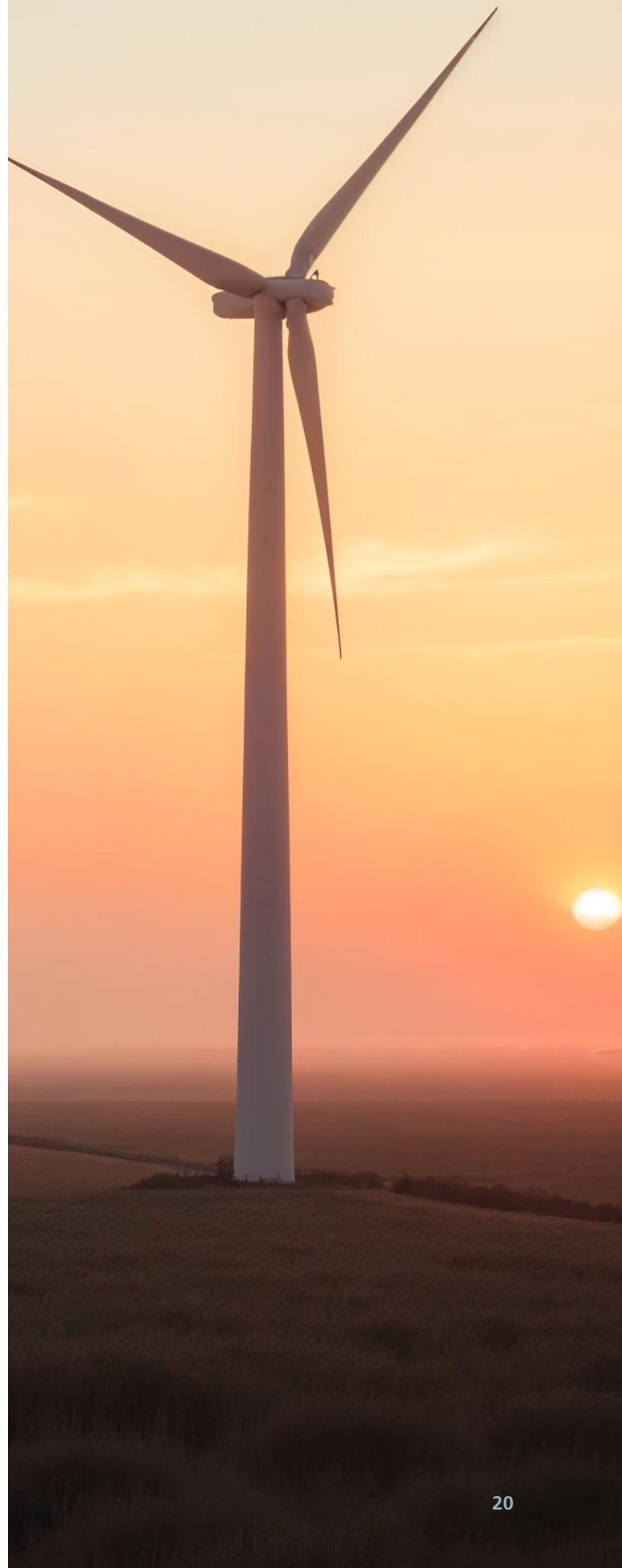
## 5.5 Allocation of Renewable Energy and Carbon Emissions

/ The allocation of sustainability aspects of electricity, such as the share of renewable energy and embodied carbon emissions, is the application area where we find both the most recent academic literature and the most publicly available practitioner documents on PFT. Traditional methods for this purpose include certification approaches such as Guarantees of Origin or Renewable Energy Certificates, which are decoupled from the physical flow of electricity (cf. Chapter 1). These do neither disclose actual numbers on emissions (as they only provide a differentiation between “green” and “grey” electricity), nor do they provide locally and temporally fine-granular data (Körner et al. 2024). In PFT approaches for allocation of renewable energy and carbon emissions, tracing results are typically derived using methods based on the proportional sharing principle (e.g., Liang et al. (2023)) and can then be multiplied by emission factors for different energy sources to obtain accurate values for units of CO<sub>2</sub>-equivalents per unit of energy (e.g., Ma et al. (2023)). In the context of renewable energies, PFT can also be applied for optimization purposes, e.g., to maximize green hydrogen production (Dudkina et al. 2022; 2024).

/ Practical implementations, aside from Eleks (2024), include emission monitors like the eCO<sub>2</sub>grid tool of 50hertz (2023) and CO<sub>2</sub>map of the University of Freiburg (2024). We note that, aside from PFT approaches, it is also possible to enhance balance sheet approaches, e.g., in a way that they provide more fine-granular data, as illustrated from researchers (e.g., Körner et al. (2024)) as well as practitioners (e.g., Agora Energiewende (2023)). In addition, a joint consideration of the balance sheet and physical properties of electricity may become even more important in the future, as it is requested by the GHG protocol for the accounting of Scope 2 emissions (i.e., emissions from purchased energy) and shown by practitioners such as Energy Track & Trace (2022) and FfE (2024).

### Application Areas

PFT approaches based on proportional sharing (both linear equation-based and graph-based) have been subject to all application areas we found in research as well as to the two areas where we found implementations in practice. Circuit theory has also been subject to research for various application areas but has - to our knowledge - not been implemented, perhaps due to performance issues.





# 6

## Conclusion and Outlook



# Conclusion and Outlook

/ In summary, the Eleks Dakar PFT approach is well grounded in foundational PFT literature, most notably the approach of Kirschen (1997). Eleks GmbH made some slight changes in their implementation, which do not affect the core idea and have also been pointed out in research (e.g., Wang et al. 2022). PFT literature has since developed other methods. Promising methods include those that do not rely on the proportional sharing principle, such as circuit theory-based PFT.

/ The Eleks Dakar PFT method is, due to the proportional sharing principle, per definition assumptive. This principle is widely discussed in literature with the aim of justifying or proving it. A definitive proof is, to date, not possible. However, some considerations, e.g., from game theory and entropy, make it seem a logical assumption. Methods that do not rely on the proportional sharing principle have other debatable assumptions (e.g., equivalent bilateral exchange), goals (e.g., optimization), or downsides such as computational complexity (e.g., circuit theory).

/ Aside from academic articles, open source literature and code on methods, utilization, and implementation of PFT in practice are scarce. To our knowledge, we are in fact the first to provide an open source white paper specifically focused on power flow tracing. This may be due to the fact that PFT methods are confidential and entities such as network operators do not have sufficient incentives to disclose them. A broader discussion of PFT approaches and applications could help improve these approaches and accelerate their implementation and adoption.

/ Most practitioner documents that we found relate to the allocation of carbon emissions (cf. Table 2). Against the pressure that governments and organizations face from society due to climate change and the large share of emissions embodied in electricity, it is logical that a variety of researchers and practitioners in electrical engineering and adjacent streams recently focus on the accounting of carbon emissions. In this light, PFT offers a great potential for significantly improving the accuracy and transparency of location-based Scope 2 carbon accounting, e.g., for fulfilling the requirements mandated by the Corporate Sustainability Reporting Directive (CSRD) as well as anticipated stricter regulations in the future. Beyond the fulfillment of regulatory requirements, an accurate data basis on emissions also provides the foundation for CO<sub>2</sub>-adaptive decisions for organizations and individuals.

/ While both literature and practice provide concepts and implementations, the data basis for a holistic and precise accounting seems to be missing for a large number of electricity grids, especially regarding low-voltage range. Further development in actual PFT methodology may not be necessary but rather related to the data input. Since it is not likely that all grid operators disclose detailed information about their power grids and even operators may not have all necessary real-time data about their grids at all times, researchers and practitioners may focus on how to integrate various data sources into their PFT approaches. This can, for example, enable more fine-grained electricity maps, illustrating detailed carbon flows and enabling CO<sub>2</sub>-adaptive decision-making for organizations and individuals. The lack of data may also be a reason why we could not find practitioners working on other application areas such as congestion management and pricing in distribution networks. Research may elaborate on the challenges for the practical application of PFT in these areas and how to overcome them to exploit its full potential.

## Call for Action

We would like to encourage researchers and practitioners to further elaborate on the following questions:

- How to provide a reliable data basis for mid- and low-voltage range?
- How to enable and combine physical and balance sheet tracing in a meaningful way (e.g., for location- and market-based Scope 2 accounting)?
- How to implement PFT methods in application areas suggested in research?



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# Appendix A - Literature Review

Author (Year)	Primary Goal	Suggested PFT Method	Main Application Area(s)
Angaphiwatchawal et al. (2024)	Enhance the accuracy of voltage impact assessments, contributing to better management of energy trading	Linear equation-based	Mitigating voltage impact
Bai and Crisostomy (2020)	Address the distribution loss allocation in peer-to-peer energy trading within a network of microgrids	Linear equation-based	Peer-to-peer energy trading
Bhand and Debbarma (2021)	Fairly allocate losses in distribution networks under a transaction energy system	Graph-based	Peer-to-peer energy trading
Budi et al. (2020)	Track power losses in electric power distribution, particularly focusing on the differences between active and reactive power losses	Optimization-based	Power losses in distribution networks
Chen and Dhople (2020)	Present circuit theory approach that provides unambiguous results consistent with the principles that describe the steady-state behavior of power networks	Circuit theory-based	Not specified
Deacon et al. (2021)	Propose a new peer-to-peer energy trading market	Linear equation-based	Peer-to-peer energy trading
Dudkina et al. (2022)	Determine and enhance the physical flow of energy from renewable energy sources to electrolyzers through the existing grid	Graph-based	Allocation of renewable energy sources
Dudkina et al. (2024)	Explore the role of batteries in maximizing green hydrogen production while adhering to the principle of additionality, ensuring that hydrogen is produced using renewable energy sources	Graph-based	Allocation of renewable energy sources
Enshae and Yousefi (2019)	Present two new algorithms for tracing the reactive power generated or absorbed by sources and loads in power systems	Linear equation-based	Allocation of reactive power flows
Jiandong et al. (2019)	Study an emergency control strategy of line overload based on power flow tracing	Linear equation-based	Line overload control
Jiang and Zhang (2021)	Investigate the physical significance of reactive power distribution and its impact on the static stability of power systems	Circuit theory-based	Voltage stability assessment
Lawal et al. (2019)	Present a method for managing congestion constraints in a hydro-thermal optimal power flow solution procedure	Graph-based	Congestion management
Li et al. (2023)	Propose a carbon flow tracing method suitable for distribution systems with distributed energy resources	Linear equation-based	Allocation of carbon emission
Liang et al. (2023)	Improve the existing calculation method of carbon emission flow in the power system	Linear equation-based	Allocation of carbon emission
Lu and Zou (2021)	Apply a complex power flow tracing method based on circuit theory to the loss allocation of bilateral transactions	Circuit theory-based	Transmission loss allocation for bilateral transactions
Ma et al. (2022)	Measure the loss of the transmission and distribution network on the network side and the carbon emissions generated by the user's electricity consumption on the load side	Graph-based	Allocation of carbon emission
Ma et al. (2023)	Propose a method for determining the nodal energy-carbon price and establish a low-carbon optimization model for energy hubs	Linear equation-based	Allocation of carbon emission
Qing and Xiang (2024)	Establish a carbon emission deduction mechanism for green electricity purchases to promote the development of the green power market	Circuit theory-based	Allocation of carbon emission
Ren et al. (2023)	Propose an improved model for tracing carbon emissions in power systems, addressing the fairness issues present in traditional methods	Linear equation-based	Allocation of carbon emission
Schäfer et al. (2019)	Analyze the hourly time-series of cross-border physical flows between European countries during 2017 and 2018	Linear equation-based	Transmission usage and import/export patterns
Shuai et al. (2021)	Improve the accuracy and scientificity of transmission allocation calculations and the economic benefit analysis of the ultra high voltage transmission network	Graph-based	Transmission network utilization
Vlaisavljevic et al. (2019)	Explain novel power flow tracing methodology called Power Flow Coloring that addresses the shortcomings of existing nodal-based methodologies	Electrical distance-based	Allocation of total redispatching costs
Wang et al. (2022)	Propose a new circuit-based approach for power tracing, known as the TISEM-based method	Circuit theory-based	Allocation of carbon emission
Wang et al. (2023)	Provide a proportional power flow tracing method to account for the carbon emission factor of electricity consumption	Linear equation-based	Allocation of carbon emission
Wang et al. (2024)	Identify key nodes in active distribution networks to enhance reliability and economic monitoring of these networks	Linear equation-based	Transparency in distribution networks
Wanghao and Paul (2019)	Address the challenges posed by bidirectional power flows in distribution grids, which complicate traditional grid modeling techniques	Circuit theory-based	Distribution grid modeling
Wu et al. (2019)	Propose a power flow tracing based load curtailment technique that efficiently restores the electrical power system during contingencies	Linear equation-based	Load curtailment
Yan et al. (2021)	Propose a real-time carbon flow algorithm for electrical power systems based on network power decomposition	Linear equation-based	Allocation of carbon emission
Yang et al. (2023)	Propose a novel flexible allocation method for carbon emissions related to transmission loss in power systems	Linear equation-based	Allocation of carbon emission
Yu (2022)	Develop an analyzing tool that combines power tracing theory with an analytics platform to study network data in the context of modern smart grids	Graph-based	Network planning for smart grids
Yu et al. (2023)	Propose a novel network loss allocation method	Graph-based	Power losses in transmission and distribution networks
Zhang et al. (2023)	Address inter-regional carbon emissions reduction in the context of low-carbon power generation	Linear equation-based	Allocation of carbon emission
Zhao et al. (2023)	Novel bidirectional loss allocation method for active distributed networks based on the Virtual Contribution Theory	Linear equation-based	Power losses in distribution networks
Zuo et al. (2024)	Propose a carbon flow tracing method based on cooperative game theory to address limitations in current carbon flow analysis methods	Game theory-based	Allocation of carbon emission

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